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# INDEX NUMBERS AND THE STANDARD OF VALUE

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### I.

The primary object of the index number is to measure or express variations in the value of money, and a little reflection will show that because of its object or end every index number involves some standard or unit of value. This standard of value may not be explicitly predicated, because on its face the index number is merely an objective, inductive investigation of prices. But when we reduce from these prices an expression of the change in the value of money, we have tacitly or explicitly set up the commodities — or a part of the commodities — to which these prices pertain, as a standard of value.

It is the belief of the writer that the index number and its peculiar problems can only be understood from the standpoint of the standard of value, and that, on the other hand, the discussion of the index number, which has now extended over more than a century, contains an important and a neglected contribution to the subject of the standard or unit of value. The object of the present paper is to supply a general theoretical introduction to the difficult and perplexing subject of index numbers

and at the same time to sum up the contribution of the statisticians to the old controversy of the standard or measure of value. I may add that in tracing the early history of the index number this object has been steadily kept in view: no attempt has been made either to describe each step or to mention the name of every contributor.

## I. THE CONSUMPTION STANDARD.

It would be difficult to determine at just what point in the development of economic thought the great importance of devising some method of measuring variations in the value of money was first recognized, but as early as 1707, in any event, the subject had been brought to the attention of Bishop Fleetwood, in an extremely interesting problem in equity, and had elicited from him the familiar dictum that the value of money depends upon the necessaries and conveniences which it will buy. "For money is of no other use," he wrote, "than as it is the thing with which we purchase the necessaries and conveniences of life."[1]

We form our concept of a definite change in the value of money from observations of changes in single prices. If a bushel of wheat costs $1 this year, whereas last year its price was $2, we say that the purchasing power of money in wheat has risen 100 per cent. But the value of money comprehends its purchasing power in all directions, over all goods consumed by the individual or group of individuals in question. Consequently, we must include in the measurement all goods consumed by the group to whom the measurement refers. Before Bishop Fleetwood's time changes in the value of money had been usually calculated from the prices of corn alone. In the crude measurement which he made on page 136 of the *Chronicon Preciosum* there were included the prices of wheat, oats, beans, ale, cloth, and meat.

But it is apparent that another factor must also be taken into consideration. The principal motive which instigated the early

[1] *Chronicon Preciosum*, ed. 1745, p. 48.

measurements was the desire to ascertain how real wages were moving; how the wage-earner was affected by changes in the general purchasing power of money. From this standpoint it is evident that changes in the prices of some goods are far more important than similar changes in the prices of other goods. If the price of corn rose 50 per cent., it was of supreme importance to the wage-earner of the eighteenth century. But a rise in the price of pepper did not effect him materially. For like reasons a fall of 50 per cent. in the price of pepper would not offset a rise of 50 per cent. in the price of corn. Evidently some method must be found of estimating the relief occasioned by falling prices in terms of the hardship occasioned by rising prices. The relative importance of the various goods must be ascertained and used before a true net result can be reached.

The first writer to take these facts into account was Sir George Shuckburgh-Evelyn, the inventor of the index number.[1] His "Estimate of the Depreciation of Money Since the Year 1050" was appended, rather irrelevantly, to a paper on "Standards of Weight and Measure." The calculation was rough and incomplete—"slovenly" some one has termed it. Like Bishop Fleetwood before him, he took occasion to apologize for meddling with such vulgar business.

Shuckburgh included in his calculations the prices of day labor and fourteen commodities. The year 1550 was selected as the standard year or "datum line," and the general price level of that year was represented by 100. No prices in their ordinary forms were given. Each price current in the year 1550 was represented by 100, and all other prices were given in the form of ratios or percentages of 100. Thus, if wheat sold for two shillings a bushel in 1550 and for six shillings in 1650, the two prices would be represented by 100 and 300, respectively. These ratios will be spoken as *price-variations* in this paper. They have been called *percentages* and *index numbers*. But the term

[1] If it may be termed an invention. His measurement is to be found in the *Philosophical Transactions for 1798*, p. 309.

*index number* is most appropriately applied to the average of all the price variations.

An index number computed as in the preceding paragraph is known as a simple or unweighted index number. The various prices are taken into account on an equal footing. If wheat rises 50 per cent. and pepper falls 50 per cent. in price, the fall of pepper exactly counterbalances the rise of wheat. Fleetwood's calculation would have been of this nature had it been carried out. Shuckburgh's calculation took account of the greater weight or importance of some prices. Twelve of the prices, as is indicated in the punctuation of the footnote,[1] were grouped together and made equal to any one of the three other prices, so that the influence of either wheat, meat, or wages in determining the result was twelve times as great as the influence exerted by any one ofthe remaining articles.

Shuckburgh's paper was followed in the early years of the present century by several valuable works bearing upon real wages and the course of prices, from Tooke and Arthur Young.[2] The latter was probably the first economist to use family budgets in estimating the change in the value of money.[3] But for the present purposes the most important writer of the first quarter of the nineteenth century is Joseph Lowe, in whose volume, *The Present State of England in Regard to Agriculture, Trade, Finance, etc.*,[4] the idea of the tabular standard appears probably for the first time. The merit of Lowe's work lies not so much in the contribution of new ideas as in the clearness with which he invested the theory implicitly contained in Shuckburgh's method. Until Lowe's time the problem had been vaguely conceived as an attempt to measure the effect of independent price variations. The net change in a vague price level was to be

[1] Wheat; meat; day labor; horse, ox, cow, sheep, hog, cock, hen, goose, butter, chese (sic), ale, small beer.

[2] TOOKE, *High and Low Prices Since 1792.* YOUNG, *An Inquiry into the Progressive Value of Money in England.*

[3] YOUNG, *Inquiry into the Rise of Prices in Europe.*

[4] I quote from the American edition of 1824.

estimated by measuring the innumerable changes in the separate prices which composed it. Lowe rationalized the whole investigation by bringing it into direct connection with the theory of value. He attempted to show that the commodities consumed by any community, taken in the relative quantities in which they are consumed, may be assumed to represent the standard unit or measure of value. Given this consumption standard of invariable value, and our problem resolves itself into the simple task of finding the variations in its total cost. Thus, if $V'$ represent the value of money in $T'$, the first epoch of time, and $V''$ the value of money in $T''$, the second epoch,

$$\therefore \frac{V''}{V'} = \frac{\text{Total price of (constant consumption list in } T')}{\text{Total price of (constant consumption list in } T'')}.$$

Modified forms of the tabular or *consumption standard*, as it will hereafter be called, have been employed in the computation of index numbers by several statisticians of the present generation. The expression of the relation of these index numbers to the ideal consumption standard will be facilitated by the adoption of the following symbols :[1]

$c_1,\ c_2,\ c_3\ \text{- - - - -}\ c_n$ = commodities.
$p_1',\ p_2',\ p_3'\ \text{- - - - -}\ p_n'$ = their respective prices in $T'$.
$p_1'',\ p_2'',\ p_3''\ \text{- - - - -}\ p_n''$ = their respective prices in $T''$.
$q_1',\ q_2',\ q_3'\ \text{- - - - -}\ q_n'$ = the respective quantities consumed in $T'$.
$q_1'',\ q_2'',\ q_3''\ \text{- - - - -}\ q_n''$ = the respective quantities consumed in $T''$.

In this nomenclature, the simple unweighted index number will be represented as follows :

$$\frac{V''}{V'} = \left(\frac{p_1'}{p_1''} + \frac{p_2'}{p_2''} + \text{ - - - - - } \frac{p_n'}{p_n''}\right) \div n. \qquad (1)$$

In the symbols adopted above allowance has been made for the fact that the actual consumption of the various commodities varies irregularly from epoch to epoch. The consumption standard requires, however, that we assume a fixed commodity or

[1] $\frac{V''}{V'}$ will be used to represent the change in the value of money. Index numbers are usually, however, given in the form $\frac{V'}{V''}$ —"the change in the price level."

*consumption list* as standard; *e. g.*: $q_1 c_1 + q_2 c_2 - - - - q_n c_n$. From which we have the *consumption index number*:

$$\frac{V''}{V'} = \frac{q_1 p_1' + q_2 p_2' - - - - + q_n p_n'}{q_1 p_1'' + q_2 p_2'' - - - - + q_n p_n''}. \qquad (2)$$

As we have no accurate statistics of consumption, (2) is usually computed from (1) in actual practice by weighting the price variations in rough accordance with their importance in consumption. It is evident, however, that in accordance with our assumption of a fixed consumption list, the exact weights would be the amounts expended upon the respective commodities in the second epoch, $T''$.

$$\frac{V''}{V'} = \frac{\frac{p_1'}{p_1''}(q_1 p_1'') + \frac{p_2'}{p_2''}(q_2 p_2'') - - - - + \frac{p_n'}{p_n''}(q_n p_n'')}{q_1 p_1'' + q_2 p_2'' - - - - + q_n p_n''}.$$

If we were computing the index number in its usual form, $\frac{V'}{V''}$, it is apparent that we should have to employ as weights the respective amounts expended in the first epoch.

The essential defect of the consumption standard is, of course, the fact that the consumption of society is not fixed. When the price of a commodity rises, we buy some substitute, and thus neutralize to a certain extent the hardship involved in the rise of price. When a commodity falls in price we use more of it, though under the new conditions and greater consumption we do not enjoy the commodity as much per unit, nor would we pay as much for it. We thus have two different standards of value: the consumption list of $T'$ and that of $T''$. Both are wrong, and, in consequence, an average or mean of the two consumption lists is usually employed. As the quantities can be averaged in an infinite number of ways, there is no limit to the number of these average standards which may be suggested. Professor Edgeworth has collected a number of these compromises in his monograph published in the *Report of the British Association* for 1887 (page 265). I quote two of these for purposes of illustration, and add another — No. 5 — suggested by a

reading of Lehr's *Beiträge zur Statistik der Preise*. So far as I can see, it is neither better nor worse than the other members of this group.

$$\frac{V''}{V'}=\frac{1}{2}\left(\frac{q_1'p_1' - - - + q_n'p_n'}{q_1'p_1'' - - - + q_n'p_n''}+\frac{q_1''p_1' - - - + q_n''p_n'}{q_1''p_1'' - - - + q_n''p_n''}\right). \quad (3)$$

$$\frac{V''}{V'}=\frac{\frac{1}{2}(q_1'+q_1'')p_1' - - - - - + \frac{1}{2}(q_n'+q_n'')p_n'}{\frac{1}{2}(q_1'+q_1'')p_1'' - - - - - + \frac{1}{2}(q_n'+q_n'')p_n''}. \quad (4)$$

$$\frac{V''}{V'}=\frac{\left(\frac{q_1'p_1'+q_1''p_1''}{p_1'+p_1''}\right)p_1' - - - - - + \left(\frac{q_n'p_n'+q_n''p_n''}{p_n'+p_n''}\right)p_n'}{\left(\frac{q'p'+q''p_1''}{p_1'+p_1''}\right)p_1'' - - - - - + \left(\frac{q_n'p_n'+q_n''p_n''}{p_n'+p_n''}\right)p_n''}. \quad (5)$$

It is needless to say that, in addition to the theoretical difficulty just mentioned, the consumption index number is beset by many practical problems arising from the impossibility of securing accurate statistics of consumption or complete statistics of prices. The latter raises the grave question whether in the actual computation of index numbers it is better to include as many price variations as possible, or to restrict the calculation to the prices of very important articles. This question, which is as difficult as it is important, will be considered in a subsequent paper. The lack of accurate statistics of consumption makes it impossible to secure the weights which the consumption index number seems to require; but this difficulty, on the other hand, does not appear to be serious. A careful comparison of weighted and unweighted index numbers calculated from the same price variations shows that the difference is usually inappreciable in comparison with the general margin of error to which all such calculations are liable. Indeed, this result might be expected *a priori*. As we know of no connection between the price variation and the importance of a commodity—the prices which rise are as likely to pertain to important commodities as those which fall, and *vice versa*—it is to be expected that the weights given to price variations below the arithmetical mean will about offset those given to price

variations above that mean, and that the weighted average will be substantially the same as the arithmetical average.

But the change in the consumption list — the variation in the standard unit — is more than a difficulty; it is an essential defect that vitiates the whole consumption standard. A change in the *per capita* consumption of a commodity is, by all theories of value, the surest indication of a change in the value of the commodity, and, in consequence, a standard of value made up of commodities whose quantities are ceaselessly changing lacks all fixity of value and is, therefore, no standard. The familiar weighted index number is merely the old "tabular standard" in operation, and both when examined closely are found defective.

To the statement that a change in quantity consumed is practically always equivalent to change in value per unit, the "practical man" would probably reply that he cares nothing about this metaphysical species of value; that all he desires is to find the variation in the "purchasing power" of money over commodities in general, assuming that a commodity is a commodity and of fixed importance in both epochs and in whatever quantity consumed; that this is what the statistician, the statesman, and the student of currency is most concerned in doing.

And in this statement we should all heartily acquiesce if it were possible to measure, even approximately, the change in the purchasing power of money so defined. But it is not possible. If we take our "practical" friend literally, it follows that money has a purchasing power over the commodity $c_1$, another purchasing power over $c_2$, a different purchasing power over $c_3$, and so on. When the prices of these commodities vary in different directions and degrees, we have a number of incommensurable price variations which, by our definition of purchasing power, cannot be reduced to an average. John Stuart Mill, for instance, would probably have taken this stand if he had not based his conclusions in this matter upon a wholly impossible case: that where a dollar buys exactly twice as much of each commodity in one time or place, as in the other. In this case the consumption standard is, of course, feasible.

But the "practical" man is usually willing to admit that in this calculation certain commodities and prices are much more important than other commodities and prices, and he proceeds to construct weights in accordance with the importance of the several commodities in consumption. The essential fallacy involved in this procedure will best be shown, perhaps, by stating the complete problem in its lowest terms. In $T'$ and $T''$, respectively, the community in question actually expends

$(q_1' p_1' - - + q_n' p_n')$ dollars for the commodities $q_1' c_1 - - + q_n' c_n$,
$(q_1'' p_1'' - - + q_n'' p_n'')$ dollars for the commodities $q_1'' c_1 - - + q_n'' c_n$.

The $q$'s and $p$'s being supposed to be known, we may designate by $A'$ etc., the expressions $\left(\frac{q_1'}{q_1' p_1' - - - q_n' p_n'}\right)$, etc., and write:

Purchasing power of the dollar in $T' = A_1' c_1 + A_2' c_2 - - - + A_n' c_n$,
Purchasing power of the dollar in $T'' = A_1'' c_1 + A_2'' c_2 - - - + A_n'' c_n$.

The index number which we desire is the ratio between the right-hand members of these equations.

But these members cannot be reduced to a simple numerical ratio as they stand, except, indeed, in the imaginary case in which $\frac{A_1'}{A_1''} = \frac{A_2'}{A_2''} = \frac{A_n'}{A_n''}$. They are composed of diverse and heterogeneous commodities which must be weighted in accordance with their importance—*i. e.*, expressed in terms of a common unit—before the index number can be secured. And the commodities have but a single attribute or quality in common—that of price or purchasing power, dependent upon their ability to satisfy human wants. Their importance is proportional, at any given time, to their prices.

But when we come to weight the commodities in accordance with their prices we encounter the essential difficulty which vitiates this method of measurement. Shall we take the price relations of $T'$, or those of $T''$, or an average between the two? We can do none of these. The importance of the commodities is not proportional to their prices in $T'$ or $T''$ because there is exactly the same reasons for using one set of price relations as the other, and yet they give different results. Both, then, are

necessarily wrong. But neither can we average these price relations, because they are expressed in terms of the monetary unit and *ex hypothesei* this unit varies between $T'$ and $T''$. The reason why this essential defect of the weighted index number is not more clearly understood is probably to be found in the lack of accurate statistics of consumption. As it is a foregone conclusion that only the roughest approximations of the weights can be secured, nobody seems to have taken the trouble to inquire whether, theoretically, there was any definite mathematical quantity to approximate.

I have expressed the problem in this manner because this is the natural and logical way of attacking it. In the actual computation of index numbers, however, the practice is usually to ascertain the individual price variations and then weight them in accordance with the importance of the commodities in consumption. But this importance can be determined only from the expenditures for these commodities in actual consumption. And here again we meet the old difficulty. Shall we use the expenditures of $T'$, those of $T''$, or an average between them? As in the preceding paragraph and for the same reasons, we cannot use any of these as weights.

For reasons similar to those given in the two preceding paragraphs, it is also impossible to average the consumption of the various commodities, as is done in the index numbers quoted on page 7. We cannot average the quantity of $c_1$ consumed in $T'$ and $T''$ because for present purposes $c_1$ is not the same thing in $T'$ as in $T''$. The purchasing power of money is determined by its command over a certain commodity list, composed of different quantities of different commodities. Each commodity, then, at any given time, represents or is a certain quantity of purchasing power. Now when we average the quantities $q'$ and $q''$, of any commodity, we assume that the purchasing power of that commodity is the same in $T'$ as in $T''$,—an impossible assumption, as the purchasing power of almost every commodity (and $c_1$ is any commodity) varies in any considerable interval of time, $T'-T''$.

There is, thus, nothing to be gained by abandoning "value" and confining ourselves to "purchasing power." The impression seems to have become general, not only among practical men, but among certain economists that metaphysical subtleties are to be avoided by dismissing the attempt to measure the change in what the Ricardians called the "real value" of money, and confining our efforts to the measurement of change in the purchasing power; or to be more practical and objective still, to the measurement of the "average variation in prices" or the "net change in the general level of prices." But as we have seen, not a single difficulty is avoided by this manipulation of words. The problem that worried Ricardo is before us still. As has been shown, there is—from the standpoint of the consumption standard at least—no average variation of prices, no net change in the general level of prices. If we pay $1,000 for twenty horses, we can usefully and intelligently say that their average price is $50. But we can no more average weights, as described above, than we can average one opera, ten strawberries, and twenty donkeys. This, I take it, is what Professor Marshall meant when he said: "A perfectly exact measure of purchasing power is not only unattainable, but even unthinkable."[1] Like Lewis Carroll's nonsense verses, the phrase "average variation of prices" sounds well, but means nothing.

## II. THE LABOR OR PRODUCTION STANDARD.

One of the gravest and most obvious practical defects of the consumption standard is its inadequacy as a standard of deferred payments. If in $T'$, A borrows $100 from B when both are making $1 a day, and prices are at a point represented by 100, how much should A repay in $T''$ when both are making $2 a day and prices are at a point represented by 95? A is as able to repay $200 as B was to loan $100, and yet according to the consumption standard B should return only $95. Imagine the reverse case, in which prices rise and wages fall, and it will be still more apparent that the consumption standard is wholly

[1] See *Third Report of the Royal Commission of Trade and Industry*, pp. 422, 423.

one-sided; fairly equitable from the standpoint of the consumer or creditor, but completely unsuited to voice or express the relative ability of the debtor or producer to repay.

An examination of pages 9 and 13 of the first volume of the Aldrich Report upon *Wholesale Prices, Wages, and Transportation* will reveal the fact that the illustration which has been used here is not a fanciful one, but based upon actual conditions as they existed at the beginning and end of the period covered by that report. During this period—measured by the consumption standard as closely as it can be applied in practice—prices fell a little while wages rose a great deal, yet the consumption standard constitutes an equitable standard of deferred payments only on the assumption that prices and wages vary in the same direction and, approximately at least, in the same ratio. It is interesting to note that the writer who first proposed the consumption or tabular standard—Lowe—understood clearly what many of his followers do not: that the consumption standard, if tenable, would apply only to a limited class.[1]

That this defect in the consumption standard should be noticed, and measures taken to devise a standard adapted to the needs and interests of producers as opposed to consumers, was inevitable. The consumers' standard came from a group of writers who were preëminently concerned with the welfare of the laborer as affected by the change in real wages, whose interests were humanitarian rather than scientific and whose standard was practical rather than theoretical; the principal commodities taken in the average quantities in which they were consumed furnished a standard sufficiently precise for their purposes. The production standard shows a similar hereditary bias. It proceeded from a body of theorists who endeavored to formulate the laws supposed to obtain in a well-simplified industrial world, and who consistently emphasized and claimed that the prosperity of the laborer depended upon the welfare of the producer. The

[1] See appendix to *Present State of England*, entitled: "How far are particular tables required for particular classes?" "A scale formed on the table in the text is adapted to very many persons in the middle and upper classes. . . . . But in regard to several of the classes currently termed productive, the question is different. . . . ."

result was the cost of production theory of value and the substitution of a single commodity—labor—for all commodities, as a standard of value.

It would hardly be profitable to thresh over the old classical controversy about the measure of real value, or to point out the delicate differences between those who accepted some form of the labor standard. As formulated by Adam Smith, the labor standard is purely subjective: a standard of disutility based upon the assumption that if there are no great variations in the time and painfulness of labor, the labor may be regarded as constant in value.[1] Unfortunately, in some respects, Smith also formulated the doctrine that in primitive society commodities will exchange in proportion to the respective amounts of labor expended in their production. From this theory it is easy, though illogical, to conclude that the value of labor is invariable at all times. As has so often been pointed out, this fact seems to have been understood by Smith. He was quite willing that his labor standard should derive whatever support it could from the more general cost of production theory, but he did not base the labor standard upon the latter theory. He went boldly over to a subjective basis and affirmed that a fixed amount of labor sacrifice is equivalent to or *is* a constant amount of value. Labor is the standard not because the ratio of exchange between two commodities tends to be the same as the ratio between the respective amounts of labor necessary to produce them, but because the value of a fixed quantity of labor is also fixed. With this postulate there is no necessity for the cost of production theory.

The classical economists, however, interpreted the labor standard in the light of the cost of production theory of value, and the inevitable result of this interpretation was its successive modification and final rejection by the best minds of the first half of the nineteenth century. Smith's dictum, "Equal quantities of labour, at all times and places, may be said to be of equal value to the labourer," entails the obvious consequence that the real

[1] *Wealth of Nations*, book i. chap. v.

wages of the laborer are of invariable value. This was so contradictory of history and everyday experience that Ricardo had no trouble in securing a general acceptance of his emendation that it was not the labor for which a commodity would exchange, but the labor incorporated or realized in it, that measured its value.[1] Then it became apparent that as in modern society goods are produced by the co-operation of labor and capital, the latter must in some way be linked with labor in the standard of value. Next it was recognized, dimly at least, that the cost of production theory of value is a theory of the causes which would regulate exchange value in an artificially simplified industrial world, and that what was required in a standard of value was a constancy in the forces or causes which regulate its value. This necessitated the further modification that the ideal standard was one of labor sacrifice and the abstinence involved in saving, and a doubt naturally arose as to the commensurability of the component parts of the standard and was argued at great length by Malthus in his *Measure of Value Stated and Illustrated*. Finally, there appeared, in germ at least, Professor Marshall's thesis that, as contemplated by the cost of production theory, labor is itself a product of labor, and that to estimate the labor incorporated in any commodity we would have to summate an indefinite series, about the early terms of which we can know nothing. By this time the discussion had reached a depth of speculative obscurity which almost justifies the contempt which the younger Mill evidently felt for it all.[2] His feeling was probably intensified by the admission of such men as Ricardo and Cournot that while we can imagine an invariable standard and "hypothetically argue and speak about it" as if we possessed it, the standard itself is unattainable.[3]

The labor standard possesses a certain plausible importance

[1] Also stated by Lord Lauderdale in his *Inquiry into the Nature and Origin of Public Wealth*, p. 26 *et seq.*

[2] *Principles of Political Economy*, book iii. chap. xv.

[3] RICARDO, *Principles*, chap. xx; and *Letters of Ricardo to McCulloch*, edited by Hollander, p. 154 *et seq.* COURNOT, *The Mathematical Principles of Wealth*, chaps. i and ii.

as a standard of deferred payments and it has been indorsed for this and other purposes by many prominent economists, including, it would seem, Professor Marshall.[1] As a standard of value it has peculiar and fatal deficiencies, but in its development and theoretical weaknesses it is essentially akin to the consumption standard.

Representing wages or the prices of the different kinds of labor by $w_1', w_2', w_n'$, in $T'$, and by $w_1'', w_2'', w_n''$, in $T''$, we have according to Adam the *unweighted labor index number*

$$\frac{V''}{V'} = \left(\frac{w_1'}{w_1''} + \frac{w_2'}{w_2''} \quad - \quad - \quad - \quad - \quad + \frac{w_n'}{w_n''}\right) \div n. \tag{6}$$

This is evidently similar to the simple arithmetical index number of prices marked (1), and has the same defect; it takes no account of the relative importance of the various kinds of labor. The difficulty may be treated as the similar difficulty was treated in the case of the consumption index number. If we regard society as a huge individual expending the same amounts of labor each year, we will have a standard labor list the various elements of which remain constant. Thus, representing quantities of labor by $q_1, q_2, q_3, q_n$, etc., we have

$$\frac{V''}{V'} = \frac{q_1 w_1' + q_2 w_2' \quad - \quad - \quad - \quad - \quad + q_n w_n'}{q_1 w_1'' + q_2 w_2'' \quad - \quad - \quad - \quad - \quad + q_n w_n''}.$$

This may be called the *weighted labor index number*. It may be obtained from the unweighted number by weighting the latter with the respective amounts expended upon the several kinds of labor in the second epoch. As in the consumption index number, $q_1, q_2, - - - q_n$ represent some mean or average of the respective quantities $(q_1' + q_1''), (q_2' + q_2'') - - - (q_n' + q_n'')$.

Mr. Bowley has used a composite labor list, made up of the various kinds of labor in the proportions in which they are on

[1] "When it (appreciation of gold) is so contrasted, and used in denoting a rise in the real value of gold, I then regard it as measured by the *increase* in the power which gold has of purchasing labour of all kinds — that is, not only manual labor, but the labour of business men and all others engaged in industry of any kind." *Final Report of the Gold and Silver Commission*, Appendix 1, p. 1. (The word "increase" italicized in the above is by mistake printed "diminution" in the Report.)

the average actually expended, in order to measure the average variation of wages.[1] Its use for this purpose need not be discussed here.

But as a standard of value it is evidently open to all the objections cited against the consumption standard and the index numbers based upon that standard. If we attempt to weight the independent wage variations according to the average expenditure upon the several kinds of labor in $T'$ and $T''$, we are confronted by the old difficulty that these expenditures are expressed in different units and hence cannot be averaged. For the same reason we cannot average the quantities of any particular kind of labor expended in T′ and T″: the quantity and productivity of the labor having changed, its value has not remained constant. Labor is like any other commodity in respect to the fact that its value is a function of its quantity. In the third place the general theory upon which this standard is based entails the conclusion that the real wages of any particular class of laborers are invariable in value. Either this is untrue, or the kind of "value" of which it is true does not concern us. If a day's labor procured $x$ commodities in 1860 and $x+y$ commodities in 1890, it is idle to say that the value of $x$ is equal to the value of $x+y$. If it procured $x$ in 1860 and $nx$ in 1890, we are quite willing to admit that its value in 1890 may not have been precisely $n$ times as great as it was in 1860, but under no tenable theory of value can it be exactly the same in 1890 as in 1860.

There is one other important theoretical defect in the weighted labor standard: as constituted, a variation of $x$ per cent. in the wages of a high-grade laborer has more influence in determining the result than a variation of $x$ per cent. in the wages of a low-grade laborer. As the foundation of this standard is labor sacrifice, this should not be the case. The variation in the wage of the sweat-shop worker toiling sixteen hours a day for a bare living is not less important than the variation in the wage of a trust company's president who receives a salary of $100 a day for his business sagacity. If, for instance, we are to use this

[1] See *Economic Journal*, vol. vi. p. 372 *et seq.*

standard as a standard of deferred payments, there seems no reason why one kind of wages should be given a greater *per capita* share in influencing the result than any other kind. Smith, Ricardo, and Malthus avoided all the mathematical difficulties of the labor standard by assuming that there were different qualities or grades of labor, and that the variation in wages would be substantially the same in all grades. This is exactly similar to the assumption upon which the consumption standard is based. It is not only untrue, but it is inadmissible as a working hypothesis because its use necessitates further violations of mathematical accuracy.

It is now generally admitted that the labor standard is a standard of cost of production, and this fact explains why it has been defended by so many writers as a standard of deferred payments: they desire to see debts adjusted, if they are adjusted at all, in accordance with the change in the general power of the community to earn money. For this purpose, however, our index number should be one of income rather than of wages, and might be represented by the formula

$$\left(\frac{I_1'}{I_1''}+\frac{I_2'}{I_2''} \; - \; - \; - \; - + \frac{I_n'}{I_n''}\right) \div n. \qquad (8)$$

where $I_1$, $I_2$, - - - - $I_n$ represent the incomes of different individuals. A similar, but theoretically more objectionable, formula has been proposed by Professor Newcomb,[1] who suggests that debts be adjusted in accordance with the movement of the national income, thus:

$$\frac{I_1'+I_2' \; - \; - \; - \; - + I_n'}{I_1''+I_2'' \; - \; - \; - \; - + I_n''}. \qquad (9)$$

This seems more objectionable than (8) because it attaches a greater importance to the variation of large incomes than that of small incomes.

If we assume that expenditure is equal to income in $T'$ and $T''$, or bears the same proportion to income in $T'$ and $T''$, (9) is equivalent to

$$\frac{q_1'\,p_1'+q_2'\,p_2' \; - \; - \; - \; - + q_n'\,p_n'}{q_1''\,p_1''+q_2''\,p_2'' \; - \; - \; - \; - + q_n''\,p_n''}. \qquad (10)$$

[1] *Principles of Political Economy*, p. 210.

This formula has been suggested by Professor Edgeworth, not—it is significant to note—as an expression of the change in the value of money, but as "a standard for deferred payments—calculated to afford to the consumer a value-in-use varying with the national affluence, after the manner of a sliding scale."[1]

But in reality the labor standard and its modifications are not suited to serve even as standards of deferred payments, because they are adjusted wholly to the interests of the productive classes, and a large and important part of the creditor class is not productive. As Professor Böhm-Bawerk[2] has so clearly pointed out, the ultimate standard of value, and *a fortiori* the ultimate standard of deferred payments, must sum up in itself the merits of both the consumption and the labor standards. It must rest secure upon its own foundation, but it must effect, at the same time, a rational compromise between the two others.

The real service of the economists who discussed the labor standard in the early part of the last century—to catch up the historical thread again—was the differentiation of *real value* from *relative* or *exchange value.* Relative value, as they used the term, meant ratio of exchange, and from this standpoint money has as many exchange values as there are goods for which it will exchange. As shown in the preceding section, it is impossible to measure the net variation in ratios of exchange except where the variation has been the same in each ratio.

In reality, when the price of a commodity changes, nothing definite can be inferred concerning the change in the value of money. The measure and the measured are both exposed to the forces which make things cheap and dear. In consideration of this fact real value was distinguished from relative value. If one unit of gold exchanged for $x$ units of bread, a logical explanation was found in the theory that one unit of gold contained $x$ times as many units of real or generic value as did one

[1] *Report of the British Association*, 1887, pp. 271, 272.

[2] *Annals of the American Academy*, vol. v. part i. p. 208.

unit of bread. The moment we assert that two dissimilar and materially incommensurable commodities possess the same value, we postulate the existence of some measurable quality or attribute common to both.

To the query, what is the common quality of all valuable goods? Adam Smith answered that it was the labor sacrifice or disutility for which it would exchange. Ricardo answered that it was the labor sacrifice worked up or realized in it. At present it is clearly understood that sacrifice is not value, disutility not utility, and that there is no necessary or fixed connection between the two. The proper answer to the query was given as early as 1833 by W. F. Lloyd, D.D., who filled the Drummond chair of political economy at Christ Church, Oxford, from 1832 until 1837.

Lloyd pointed out very clearly what has since been explained so minutely by Jevons, Böhm-Bawerk, and others, that the value of a commodity depends upon the marginal intensity of the want it satisfies; that want is satiable, and that, in consequence, value depends largely upon the quantity of the commodity consumed. In his "Lecture on the Notion of Value as Distinguishable, not only from Utility, but also from Value in Exchange," Lloyd explains the gradual satiability of any want, and says: "It is on want thus estimated that value depends. . . . . The gratification derivable from the use of an object must be taken to be equal to the want of it, thus estimated; and the value, properly speaking, is the feeling of affection or esteem for the object, arising from a sense of the loss of the gratification contingent on the loss of the object. . . . . In its ultimate sense, then, the term undoubtedly signifies a feeling of the mind, which shows itself always at the margin of separation between the satisfied and unsatisfied wants. . . . . As I have explained the idea, it (value) consists in the real importance of an object to the person who possesses it." These words sound very familiar to readers of Böhm-Bawerk and others of the Austrian school. They have been taken from a volume of lectures by Lloyd published at Oxford in 1837.

## III. THE MODERN INDEX NUMBER.

As in the case of the consumption and labor standards, the modern index number was a product of peculiar economic conditions existing at the time of its appearance. In the early years of the present century the principal concern of practical economic thinkers was about the wage-earner and the condition of the poor. With the passage of the reform acts and the repeal of the corn laws the predominance of interest passed to another problem. Following the report of the Bullion Commission and the resumption of specie payments by the Bank of England, a deep interest in the quantity theory of money had been aroused, which was only intensified by the passage of the Bank Act of 1844. For many years thereafter measurements of variations in the value of money were made chiefly for the purpose of testing the quantity theory of money, and this new *motif* showed itself in the methods of investigating changes in the value of money which were employed.

If the only (or the chief) cause of the variation of prices is a sudden and marked change in the quantity of money in circulation, we might expect to see all prices vary alike both in amount and direction. In the dispute between the adherents of the currency principle, and in investigating the effects of the influx of Californian and Australian gold, it became necessary to discover the cause of variations in prices. If the cause was on the money side it might be expected to show itself, as noted, in a certain uniformity of variation. If the separate price-variations were completely sporadic, however, a strong presumption was raised against the cause of variation having proceeded from the money side. In either event, the collection of price-percentages having been made, it was convenient for purposes of expression and illustration, to add them, take an arithmetic mean, and call the result the net variation in the general level of prices. The vital phenomena were the separate price variations. Any combination of these as a quantitative expression of the average amount of change was incidental. The index number appeared as a by-product.

Most of the investigations into the course of prices at this time were thus examinations into causes rather than measurements of actual changes. Porter introduces his index number in a discussion of the question, "whether under the régime of a circulating medium convertible at pleasure into gold, any issue of paper can be made and kept out to an excess that will tend to raise the general prices of goods."[1] Newmarch in his original investigation even neglected to compute a total index number.[2] Jevons himself was as much interested in proving a casual relation between money and rising prices as in measuring correctly the amount of the rise. The chief concern being with the individual price variations, there was no pressing reason to obtain the weights necessary to make the simple index number conform to the consumer's index number. The great object was to prove a uniformity in the price variations, and it is plain where this uniformity exists weighting is unnecessary. There was, thus, a return to the unweighted index number. By most writers of the time the latter was regarded as a mere approximation of the weighted average. But Jevons placed it upon a new and independent basis.

If Evelyn was the inventor of the index number, Jevons was its popularizer. The large amount of literature which has appeared on this subject, particularly in Germany and England, was mainly inspired by Jevons's pamplet entitled *A Serious Fall in the Value of Gold Ascertained*, etc.[3] Together with a great talent for the exposition of abstruse subjects, Jevons possessed in an unusual degree the faculty of feeling and communicating a decent respect for the importance of any subject upon which he wrote. He made of this essay one of the most remarkable chapters in economic literature.

The explanation of this fact is to be found in the contradictory character of the essay. It was a peculiar mixture of truth and error. Jevons began by taking sides with Lowe and

[1] PORTER, *Progress of the Nation*, p. 425.

[2] NEWMARCH, *Journal of the Statistical Society*, 1859, p. 96 *et seq.*

[3] See *Investigations in Currency and Finance*, p. 14 *et seq.*

the consumption standard. Value, he asserted, "is a vague expression for potency in purchasing other commodities, and if gold has become less potent with respect to some and not less potent with respect to others, it has fallen in value."[1] But although he gave a formal assent to the consumption standard, he made no attempt at weighting the simple index number in order to make it conform to the consumers' index number. "Ought we to take all commodities on an equal footing?" he asks. "Ought we to give most weight to those which are least intrinsically variable in value? Ought we to give additional weight to articles according to their importance, and the total quantities bought and sold? The question, when fully opened, seems to be one that no writer has attempted to decide—nor can I attempt to decide it. I regard the fall of value as conclusively proved, although the exact nature of the problem is left amid the obscurities of economic science in general."[2]

Jevons gave no categorical answer to his query about weighting. But the device was omitted and there is no doubt that this omission was a deliberate answer to the problem which he had stated. But, unfortunately, he did not or could not explain his action. He accepted the consumption standard, which requires weighting and the arithmetic mean, and then computed an unweighted number with the geometric mean. Had he explained his introduction of the geometric mean his essay would probably have escaped the searching criticisms of Forsell, Laspeyres, Drobisch, and others. But the arguments which he used to defend his course were riddles rather than reasons. It is these riddles which have most effectively contributed to the study and discussion of the proper method of computing index numbers.

Long after the publication of Jevons's pamphlet these riddles were successfully answered by Professor Edgeworth, who showed the usefulness of Jevons's method by interpreting it in the light of the theory of probability and the statistical studies of Quetelet. Jevons's method, as has been just said, differs from the

[1] *Investigations in Currency and Finance*, p. 20. [2] *Ibid.*, p. 21.

ordinary consumption index number by the absence of weighting and the use of some other than the arithmetic average. But the important difference really lies in the different interpretations put upon the price variations in the two methods. This difference will best be shown, perhaps, by an illustration.

Suppose that we had to ascertain the weight of a very large and heavy mass by means of a balance which was difficult to operate and known to be erratic and inaccurate. Two methods of procedure might be followed. We might divide the mass into a large number of particles, weigh these, and obtain our answer by adding the weights of the several particles arithmetically. This method would be somewhat analogous to the consumption index number. On the other hand, if the error of the scale was not known to be of a specific kind, we might weigh the whole mass a large number of times and take as our answer the average of the several readings of the scale. In the latter case each reading would be an independent answer in itself, no weights would be employed, and the geometrical mean, in accordance with previous experience in such work and with certain accepted scientific principles, would be preferable to the arithmetic mean. The latter operation is more closely analogous to the index number of Jevons and indicates clearly the different rôle of the price variations in the two methods. To Jevons each price variation was an independent and, before the average was computed, an equally valid answer to the question: What has been the change in the value of money? In the consumer's index number, however, each price variation is supposed to affect the value of money differently and the final answer must be obtained by summating the several variations after they have been weighted in accordance with their importance.

Professor Edgeworth has on several occasions shown very clearly the theoretical superiority of the geometric over the arithmetic mean, for the purpose of averaging price variations.[1] The question is really one of "better or worse," however, not one of "right or wrong," and it need not concern us here.

[1] See EDGEWORTH in the *Economic Journal* for March 1896, p. 137.

Practical experience with index numbers computed from the same data by different means shows that the differences are usually inappreciable in comparison with the possible errors introduced by defects in the statistical data. Jevons's method may thus be considered as the simple unweighted average of price variations, in a new interpretation.

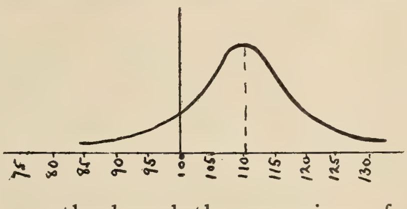
The unweighted index number is justified in several interpretations of the problem under consideration. In the first place we may take the case of those economists who refuse to admit that money has a general or homogeneous value.[1] In their view, money has one value in $c_1$ another value in $c_2$, another in $c_n$ etc., and these values are heterogeneous. If prices vary differently the "values" of money have varied differently, and strictly speaking that is all that can be said: each price variation stands by itself as an independent and complete measure of one change in the value of money. Any combination of these percentages is meaningless, but a fictitious average or type may be secured which will serve to facilitate discussion, where quantitative accuracy is not demanded, by saving words. It is merely "a compendious way of stating facts, the full expression of which would be tedious and inconvenient."

The case in which the unweighted average is most clearly applicable is that in which a sudden emission of paper money or some other cause affecting confidence in the currency has produced a general and widespread variation of prices. Owing to the nature of this cause, the variations in prices would tend to be the same, and speaking absolutely the change in the price of a single commodity would show the change in the value of money. Practically, however, if the interval of measurement $T'-T''$ were not exceedingly short, we should be forced to take an average of a large number of price variations in order to eliminate or destroy the effect of accidental deviations from the mean type. And in this case we should use the average not as a mere dialectical device for saving words, and not as an end in itself, but as an instrument to elicit and express that variation

[1] *Cf.* WALKER, *Money in its Relation to Trade and Industry*, p. 30 *et seq.*

which occurred with the greatest frequency—the variation caused by the influx of new money. Moreover, if a great and preponderant majority of the price variations were of the same amount, while the deviations from this common variation were equally distributed above and below it, we should admit that "the most frequent variation" expressed, by inversion, not only the change in the value of money due to the specific cause assumed, but the net change in the value of money resulting from all causes.

Professor Edgeworth, in a series of brilliant contributions, has made this field of inquiry all his own[1]. The *quaesitum* of the process now under consideration may be described in his words: "This type of mean variation may be generally defined as that figure which would be presented most frequently if we were to continue indefinitely the long series of price ratios, or at least that return in whose neighborhood the greatest number of these statistics cluster. It is, in other words, the Greatest Ordinate of the complete curve, or the highest column of the rectilinear diagram which represents by its abscissa ratio between the prices of two compared epochs and by its ordinate the frequency with which that ratio would be returned if the statistics were extended over every region of industry which is subject to independent fluctations." Such a type is exhibited in the subjoined curve, where abscissae represent the different price variations and the ordinates represent (or vary with) the number of times the corresponding price variation occurs. If the price variation when plotted arranged themselves as shown below, we might confidently announce 110 as the index number of prices desired.




Accepting the maximum ordinate as the *quaesitum* of this method, it becomes necessary to inquire into the validity of the method and the meaning of the result in a more complicated case, that in which the origin

[1] See *Reports of the British Association*, 1887, p. 281 *et seq.;* 1889, p. 156 *et seq.;* and *Journal of the Royal Statistical Society*, June 1888.

of the variation in prices is obscure and the interval $T' - T''$ long enough to permit the operation of all those industrial forces such as invention, combination, discovery of new sources or restriction of old sources of raw material, which cause variations in the prices of commodities. This is the case with which we actually have to deal.

Here again the validity of Jevons's method to elicit and express the change in the value of money proceeding from the money side might be defended on the grounds that price variations proceed from changes in the quantity of money in circulation and from changes in the conditions of production — "changes from the side of money" and "changes from the side of commodities"— that the change from the money side would be a uniform change, those from the side of commodities sporadic and irregular; and that the maximum ordinate — or the mean of the price variations best calculated to represent the maximum ordinate — would be an accurate expression of the change in the value of money arising from changes in the quantity of money in circulation.

If this theory be tenable, the result is so circumscribed as to be of little value. Index numbers are not computed with the idea of determining that part of the change in the value of money which has proceeded from a given cause. What we desire is the net change in the value of money. The ideal of the consumption standard is correct enough.

But the theory is not tenable: it errs by reason of excessive simplicity. Ordinarily changes in the quantity of money do not affect prices uniformly and would not affect them uniformly if invention, combination, and all the other industrial forces which affect prices were inoperative. A large influx of metallic money, for instance, affects prices not so much by depressing the general desire for or estimation of money as by entering subtly into the veins of trade, stimulating production, and supporting a large number of enterprises which otherwise would have foundered. In a large number of industries it intensifies the law of increasing returns and *pro tanto* operates to diminish

prices. In a smaller, but very important, number of industries it intensifies the operation of the law of decreasing returns and *pro tanto* operates to increase prices. Under ordinary circumstances, therefore, it seems impossible to use Jevons's method, as I believe he thought it could be used, to measure the variation in the value of money due to the change in the volume of money.

The immediately preceding paragraphs will suggest the answer to the two queries with which we are at present concerned. Whether this process be at all applicable, and whether, if applicable, it yield as a result the net variation in the value of money or something else, depends upon the symmetry of the curve of price variations. In order that the maximum ordinate should represent the net variation in the value of money, the curve would have to be exactly symmetrical; the price variations below the variation of greatest frequency would have to be exactly equal in number and importance (or in both combined) to the price variations above it. No one who has given a moment's thought to the nature and cause of price variations in ordinary periods would have the slightest expectation of finding this symmetry in a curve of price variations, and, as a matter of fact, we do not find it, or anything like it. The operation of the two great laws of increasing and decreasing returns, and the fact that prices may (theoretically) rise indefinitely while they cannot fall more than 100 per cent., are practically sufficient in themselves to destroy the possibility of a symmetrical distribution of the price variations. The unweighted index number, then, furnishes neither the net variation in the value of money nor the variation arising from the money side. What it does furnish is a broad general type of price variations, the usefulness of which is very much less than that of the net variation and something more than that of the fictitious average described on page 24.

But, as stated above, not only the meaning of the result but the general applicability of the process depend upon the symmetry of the curve of price variations. It is not sufficient that one (set of) price variation(s) should be in a majority; it must

be in a large majority, and as more and more prices are included within the scope of the investigation the curve must become smoother as it recedes from the apex, and the slope of the curve steeper: "it is a characteristic of the things with which probability is concerned to present, in the long run, a continually intensifying uniformity."[1]

Just what degree of symmetry in the curve of price variations is necessary to justify the employment of this process is a question which the writer is not competent to treat. The following reasons are given, however, for the doubt that is in him concerning the suitability of this process to elicit even a mean type of price variations with the requisite qualifications.

1. Changes in the volume of money can be expected to produce a general or uniform variation in prices only when the currency is an inconvertible paper currency and the interval of measurement very short. This is a case which seldom occurs.

2. There are constantly at work two great sets of causes, in addition to the one mentioned above, which tend to influence prices in different directions. The first, consisting of competition, invention, increase of capital, betterment of methods and all the causes known loosely as industrial improvement, operate to reduce prices. The second, including industrial combination, possibly, exhaustion of sources of supply, increase of indirect taxation, and pressure of population where the source of supply is fixed, *e. g.*, urban land, operate to increase prices. Thus, whatever the nature of the influence exerted by changes in the supply of money, it must be assisted by one set of important forces and resisted by another. As a consequence — and this is particularly true where the causes from the money side are operating to increase prices — we are not unlikely to get a curve with a double apex, showing two general types of price variations, the one only a little less "mean" than the other.

3. This possibility is indicated in the following table showing the frequency of the various price variations in each of the

[1] VENN, *Logic of Chance*, third edition, p. 455.

thirteen years, 1879–1891. The price variations or ratios, which are given as percentages of the respective prices in 1860, were counted in blocks of ten — a variation of 81, for instance, having been counted with one of 88 as a member of the group 80–90. The table is based upon returns given in the *Aldrich Report on Wholesale Wages, Prices, and Transportation*, Vol. I, pp. 30–52, the price variations being taken as they are there given, except that the twenty-seven varieties of pocketknives distinguished in the *Report* have been grouped and treated as a single commodity. The period, 1879–1891, was selected simply because it is the only period subsequent to 1860, and covered by the *Aldrich Report*, in which the currency of the United States was on a metallic basis.

The significance of these data will most readily be seen by plotting the corresponding curves. However, from the figures themselves it is apparent that in the five of the thirteen years—1879, 1880, 1882, 1885, 1886—there were subordinate movements sharply distinguished from the main movement of prices, yielding, when plotted, curves with two or more apices. The process of grouping employed here is itself equivalent to a large degree of "smoothing," and I see no reason to believe that in any year the completed set of price returns would yield a smoother or more symmetrical curve than the one given by the prices treated — nearly 200 in each year. The point has been made by Dr. Venn and Professor Edgeworth — and it seems well taken — that some average of the representative list of price variations is likely to approach the maximum ordinate of the complete set of price variations more closely than the maximum ordinate of the representative list. But this does not mitigate the necessity of basing our inferences about the general contour of the ultimate or complete curve of price variations upon the incomplete curve which we have.

To sum up the results of this and preceding sections, we may say that there is not only some doubt as to the adaptability of price statistics to this method of treatment, but the irregularity and assymetry of the curve of price variations make it

TABLE SHOWING THE FREQUENCY OF THE VARIOUS PRICE VARIATIONS FOR THE YEARS 1879-1891, INCLUSIVE.

| | 0 per cent. | 20 per cent. | 30 per cent. | 40 per cent. | 50 per cent. | 60 per cent. | 70 per cent. | 80 per cent. | 90 per cent. | 100 per cent. | 110 per cent. | 120 per cent. | 130 per cent. | 140 per cent. | 150 per cent. | 160 per cent. | 170 per cent. | 180 per cent. | 190 per cent. | 200 per cent. | 500 per cent. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1879 | 0 | 1 | 3 | 1 | 9 | 22 | 18 | 47 | 16 | 25 | 18 | 14 | 7 | 4 | 5 | 2 | 0 | 0 | 1 | 4 | |
| 1880 | 0 | 1 | 2 | 2 | 2 | 7 | 14 | 30 | 22 | 37 | 25 | 22 | 7 | 3 | 5 | 2 | 5 | 0 | 1 | 5 | |
| 1881 | 0 | 1 | 1 | 2 | 6 | 6 | 18 | 22 | 37 | 30 | 22 | 16 | 11 | 4 | 1 | 6 | 3 | 2 | 1 | 6 | |
| 1882 | 0 | 1 | 1 | 2 | 5 | 11 | 8 | 25 | 21 | 41 | 23 | 14 | 7 | 2 | 7 | 4 | 4 | 1 | 4 | 7 | |
| 1883 | 0 | 2 | 0 | 3 | 3 | 13 | 18 | 18 | 33 | 30 | 22 | 15 | 5 | 2 | 4 | 5 | 3 | 4 | 2 | 7 | |
| 1884 | 0 | 2 | 0 | 4 | 10 | 17 | 21 | 29 | 34 | 21 | 19 | 14 | 4 | 4 | 3 | 6 | 6 | 1 | 0 | 5 | |
| 1885 | 1 | 1 | 2 | 6 | 15 | 21 | 26 | 25 | 32 | 18 | 13 | 11 | 5 | 4 | 3 | 4 | 3 | 1 | 0 | 4 | |
| 1886 | 1 | 1 | 3 | 4 | 13 | 19 | 32 | 33 | 22 | 23 | 9 | 8 | 4 | 4 | 2 | 6 | 0 | 2 | 0 | 1 | |
| 1887 | 1 | 1 | 2 | 9 | 15 | 21 | 36 | 25 | 22 | 17 | 7 | 12 | 6 | 2 | 3 | 3 | 6 | 2 | 1 | 5 | |
| 1888 | 1 | 1 | 2 | 10 | 9 | 16 | 36 | 26 | 22 | 20 | 15 | 16 | 11 | 1 | 3 | 3 | 1 | 4 | 0 | 6 | |
| 1889 | 1 | 1 | 4 | 7 | 13 | 24 | 27 | 35 | 13 | 18 | 13 | 11 | 5 | 7 | 2 | 3 | 4 | 1 | 2 | 5 | |
| 1890 | 1 | 1 | 4 | 8 | 16 | 21 | 30 | 27 | 17 | 18 | 11 | 9 | 7 | 4 | 7 | 2 | 2 | 2 | 0 | 6 | |
| 1891 | 1 | 4 | 5 | 7 | 16 | 17 | 30 | 26 | 23 | 19 | 13 | 11 | 7 | 4 | 4 | 4 | 2 | 1 | 0 | 5 | |

extremely improbable that the mean variation will in any particular case be a close approximation of the net variation. As a matter of fact, however, there is no "net variation," as this term is generally understood. The reasons for this have been previously given in the treatment of the consumption standard, and they amount essentially to the important fact that we cannot treat goods which are consumed in different quantities as of invariable value. The attempt to do it is not only contrary to all recognized theories of value, but as a simple proposition of statistics it results in producing a number of formulæ or index numbers which are different, yet nevertheless have equally valid claims for selection. It might almost be laid down as a general proposition of logic where such a condition results from a course of deduction in the successive links of which no error can be discerned, that a mistake has been made in the original premises.

The foregoing constitutes what might be called the ultra theoretical case against the ordinary index number. In addition, however, the standards which treat ordinary commodities and labor as invariable in value, irrespective of the quantity in which they are consumed or purchased, are beset by the important practical difficulty that they represent the interests of a

limited class, either those of the consumers or those of the producers—and these classes do not coincide. There is a practical as well as a strong theoretical and historical necessity for some standard of deferred payments logically intermediate between the consumption and labor standards.

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# INDEX NUMBERS AND THE STANDARD OF VALUE.

## II.

In the preceding paper a slight sketch was given of the development of the index number up to and including the work of Jevons on this subject. It is not intended to prolong this sketch. The literature of the index number, after Jevons, must be studied at first-hand, although it is hardly an exaggeration to say that almost every writer since Jevons — Laspeyres, Drobisch, Lehr, Pierson, Oker, and Padan among them — has begun with Jevons's riddles and finished by condemning his standard because of its obvious conflict with the requirements of the consumption standard. Even Professor Edgeworth, who was the first to explain the merits of Jevons's work in this line, began by criticising it adversely.[1] Thus, the modern discussion of the index number not only began with Jevons, but in the present paper — which is devoted to an exposition of what may be called, relatively speaking, the "best standard of value" — it will be seen that the hints and suggestions embodied in Jevons's work have not yet been thoroughly exhausted.

It will be remembered by those familiar with Jevons's work on this subject that he arbitrarily altered some of the price variations which he considered excessive. The specific offense — to which he himself calls attention — consisted in toning down certain variations in the prices of cotton, hemp, flax, and tallow, which had been abnormally raised by the effect of war in the United States and Russia. Jevons's action cannot be considered as intrinsically wrong or unfair. Neither can it be unequivocally indorsed. Everything, of course, depends upon what stage in the proceedings we are to give ourselves over into the hands of the science of probability. If, as Jevons intimated was the

[1] See *Journal of the Statistical Society*, December, 1883.

case, we are to assume ignorance as soon as the price variations have been recorded, and are to take all variations on an equal footing, then he was guilty of breaking the prearranged rules of the game. If, on the other hand, we possess the criteria by which to adjudge one set of returns inadmissible, why not use our knowledge to extend the process of elimination until only the correct variation (or set of variations) remain? To be able to decide that one set of returns does not measure the change in the value of money, implies the possession of at least a vague notion of what constitutes the true standard of value. Why not clarify this notion and extend the field of actual knowledge instead of summoning to our aid the weapons of ignorance, powerful though they be? Just as in every physical measurement the limitations of human sensibility and the imperfection of mechanical instruments necessitate the employment of averages and the logic of probability, so in the measurement of value we will always be compelled to use these devices at some point in the proceedings. But this does not relieve us of the necessity of excluding every observation or return that is known to be erroneous.

The principle of exclusion employed by Jevons, however, can scarcely be said to furnish the criterion by which to decide whether a given commodity may be included in the measurement or not. He modified the price of cotton because his object was to ascertain the change in the value of money due to the increase in the precious metals, and the extraordinary variation in the price of cotton was confessedly due to other causes. "Our duty," he wrote, "as regards fluctuations due to changes of supply of commodities is to have nothing to do with them, but to eliminate the effects from our inquiry as soon and as completely as possible." [1]

Such an explanation is about as mysteriously irritating as his riddles respecting the proper mean to be employed in averaging price variations. Nevertheless, it rests upon a broad basis of truth, and finds acceptance not only in economic theory

[1] *A Serious Fall*, etc., pp. 25, 26.

but in the common sense of the business man. The latter would be quite unwilling to include in the standard of value a commodity whose supply was twice as great in $T'$ as in $T''$, yet this is what he often does when he accepts an ordinary index number as an expression of the change in the value of money. The classical theory of money, to refer to the science itself, incorporated the same idea in the axiom "value depends upon demand and supply." If the supply were visibly and confessedly different in the two epochs there would be good grounds for the belief that the value of the commodity had changed.

But the truth, towards which Jevons was groping, was a more important one than that incorporated in the negative statement that a commodity whose supply has changed in a large degree cannot be included in a standard of value. What we want to know is what constitutes a constancy of value. The reply to that question was given later by Jevons himself,[1] and in clearer and more precise terms by those writers who have developed the theory of marginal utility which he propounded.[2] Value is a function of quantity consumed: the "ultimate" or "best" standard of value is a restricted consumption standard, each commodity of which is consumed in the same quantity in $T'$ as in $T''$. In anticipation of the objections of those readers who admit that marginal utility is a function of quantity, but deny that the same is true of value, it may be stated that reasons will be given hereafter to show that in the measurement of changes in the value of money, at least, no real distinction exists between value and utility or marginal utility.

Since the appearance of the Austrian theory of value a number of monographs have been published in which the standard of value is discussed from the standpoint of the marginal utility theory of value, and at least two utility standards have been suggested which require notice here. An examination of their

[1] *Theory of Political Economy*, p. 165 *et passim.*

[2] See MARSHALL, *Principles*, Appendix, Note XII; and IRVING FISHER, *Mathematical Investigations in the Theory of Value and Prices*, pp. 18, 87, 89.

rationale will demonstrate how perfectly natural, even commonplace, a standard of utility is.

The merit of first proposing a standard of utility adapted to the measurement of changes in the value of money must, so far as I am aware, be ascribed to Dr. Julius Lehr.[1] Dr. Lehr describes the fundamental theory upon which the whole group of utility measurements rest, as follows:

Um nun die Aenderungen des Geldwerthes bemessen zu Konnen, mussen auf einem bestimmten Gebiete (Land, Provinz) Alle Waaren und Leistungen, für welche Preise gezahlt worden sind, berücksichtigt werden. Hierauf sind dieselben auf ein gemeinschaftliches Masz zu bringen. Eine Handhabe hierfür bietet die jeweilige Gleichwerthigkeit. Man Kann nämlich diejenigen Mengen von Waaren und Leistungen einander gleich setzen, welche als gleich werthig zu betrachten sind. Als gleichwertig aber haben wir in unserem Falle, in welchem es sich nur um die Begriffe Preis, Durchschnittspreis, Marktpreis handelt, die jenigen Mengen anzusehen, für welche gleich viel gezahlt wird. Ist der Preis eines Hektoliter Wein=60 Mark, der eines Festmeter Buchenscheitholz=10 Mark, so sind 6 Festmeter Holz einem Hektoliter Wein gleich zu setzen. Für eine Mark erhalten wir dann $\frac{1}{60}$ Hektoliter Wein, ebenso auch $\frac{1}{10}$ Festmeter Holz. Diese Mengen wollen wir als je eine "Genusseinheit" bezeichnen, ein Begriff, der in Folgender Weise zur Berechnung der Veränderungen des Geldwerthes benutzt werden kann.[2]

Expressed in our notation Dr. Lehr would say that at any given time $\frac{1}{p_1}c_1$ or $\frac{1}{p_2}c_2$ - - - - or $\frac{1}{p_n}c_n$ each yields or is equal to a unit of utility; or expressing the same idea in another way: utility of $c_1$ : utility of $c_2$ : utility of $c_n$ : : $p_1 : p_2 : p_n$. But at this point we are met by the difficulty that the prices change. At any given time the utility of various commodities is proportional to their prices, but what is to be done when these prices change?

Dr. Lehr met this difficulty by computing a formula for the average price of a commodity. If in $T'$, $q_1'$ units of $c_1$ are sold at $p_1'$ per unit, while in $T''$, $q_1''$ units of $c_1$ are sold at $p_1''$, per unit, then the total price of $(q_1' + q_1'')$ units of $C$ is $(q_1'p_1' + q_1''p_1'')$, and on the average of $T'$ and $T''$ one unit of $c_1$ costs $\frac{q_1'p_1' + q_1''p_1''}{q_1' + q_1''}$. This amount, Dr. Lehr declares, is the true aver-

[1] *Beiträge zur Statistik der Preise* (Frankfort, 1885).

[2] *Ibid.*, pp. 37, 38.

age price of $c_1$ because this price multiplied by the quantity of $c_1$ actually bought in $T'$ and $T''$ will give the amount actually spent for $c_1$ in $T'$ and $T''$.[1] In consequence:

ut. of $c_1$ : ut. of $c_2$ : ut. of $c_n$ ::

$$\frac{q_1' p_1' + q_1'' p_1''}{q_1' + q_1''} : \frac{q_2' p_2' + q_2'' p_2''}{q_2' + q_2''} : \frac{q_n' p_n' + q_n'' p_n''}{q_n' + q_n''} .$$

If in $T'$ $(q_1' p_1' - - - + q_n' p_n')$ dollars buy $(q_1' c_1 - - - + q_n' c_n)$, and in $T''$ $(q_1'' p_1'' - - - + q_n'' p_n'')$ dollars buy $(q_1'' c_1 - - - + q_n'' c_n)$,

then $$\frac{V''}{V'} = \frac{q_1'' c_1 - - - + q_n'' c_n}{q_1' c_1 - - - + q_n' c_n} \times \frac{q_1' p_1' - - - + q_n' p_n'}{q_1'' p_1'' - - - + q_n'' p_n''} .$$

Substituting for $c_1$, - - - $c_n$ their respective values as determined by their average prices, we get Dr. Lehr's formula:

$$\frac{V''}{V'} = \frac{\left(\frac{q_1' p_1' + q_1'' p_1''}{q_1' + q_1''}\right) q_1'' - - - + \left(\frac{q_n' p_n' + q_n'' p_n''}{q_n' + q_n''}\right) q_n''}{\left(\frac{q_1' p_1' + q_1'' p_1''}{q_1' + q_1''}\right) q_1' - - - + \left(\frac{q_n' p_n' + q_n'' p_n''}{q_n' + q_n''}\right) q_n''} \times \frac{q_1' p_1' - - - + q_n' p_n'}{q_1'' p_1'' - - - + q_n'' p_n''} .$$

The validity of Dr. Lehr's method depends upon the average he employed. If his formula is proposed as a mere empirical approximation, it may be credited with that degree of validity appertaining to a mean between two erroneous results, whose respective errors are not known to be of an opposite kind. From a theoretical standpoint there is no superior virtue—in fact, no virtue at all—in the average he used.

According to Dr. Lehr, commodities have an importance proportional to their prices. If the price of $c_1$ in $T'$ is $p_1'$, $\frac{1}{p_1'}$

[1] *Beiträge*, p. 39. By similar reasoning we might say that the average quantity of $c_1$ consumed in $T'$ and $T'''$ is $\frac{q_1' p_1' + q_1'' p_1''}{p_1' + p_1''}$. Weighting the prices with these average quantities we get the following consumers' index-number, given as (5) on page 7 of the preceding article (JOURNAL OF POLITICAL ECONOMY, December, 1901).

$$\frac{V''}{V'} = \frac{\left(\frac{q_1' p_1' + q_1'' p_1''}{p_1' + p_1''}\right) p_1' - - - - + \left(\frac{q_n' p_n' + q_n'' p_n''}{p_n' + p_n''}\right) p_n'}{\left(\frac{q_1' p_1' + q_1'' p_1''}{p_1' + p_1''}\right) p_1'' - - - - + \left(\frac{q_n' p_n' + q_n'' p_n''}{p_n' + p_n''}\right) p_n''} .$$

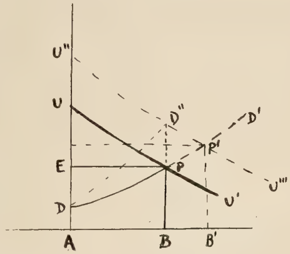
units of $c_1$ contains one unit of utility, $c_1$ contains $p'$ units of utility, and its importance may be represented by $p'$. Similarly, the importance of $c_1$ in $T''$ may be represented by $p_1''$; but $p_1'$ and $p_1''$ cannot be added, since they are incommensurable. We know that the importance of $c_1$ in $T'$ is $p_1'$, because its money value is $p_1'$. We also know that the importance of $c_1$ in $T''$ is $p_1''$, because its price is $p_1''$. But unless the value of money has remained the same $p_1'$ and $p_1''$ are expressed in different units. In other words, Dr. Lehr has assumed the value of money to be the same in the two epochs, in order to measure the change in the value of money by another process. Dr. Lehr seems to have been led into error by forgetting that prices represent relations or ratios between amounts of utility and not absolute amounts of utility. In $T'$ the utility of $c_1$ may be represented by $p_1'$, and in $T''$ the utility of $c_1$ may be represented by $p_1''$, but we know nothing of the ratio $\frac{\text{ut. of } c_1 \text{ in } T'}{\text{ut. of } c_1 \text{ in } T''}$. The "genusseinheit" of $T'$ is not the "genusseinheit" of $T''$. Prices represent ratios between amounts of utility, not positive amounts of utility.

Lehr's formula is valid in two special cases, in both of which our problem might easily be solved without the unwieldy formula which he has given. It is evident that the process is correct, where ut. of $c_1$ in $T'$ = ut. of $c_1$ in $T''$; ut. of $c_n$ in $T'$ = ut. of $c_n$ in $T''$. The formula will also be correct where $\frac{p_1'}{p_1''} = \frac{p_2'}{p_2''} = \frac{p_n'}{p_n''}$. When the utility of commodities remains unchanged from $T'$ to $T''$, or when the variation has been the same in all prices, Lehr's formula holds good. The only instance in which either of these phenomena can occur is where the variation in prices lies wholly on the side of money. Lehr's method agrees with all its predecessors in being applicable only in that one special instance, which never occurs.

What we have called the weighted labor standard has been again proposed as a measure of marginal utility by Professor Clark.[1] If units of labor-time be measured along $AB$, and the

[1] *Yale Review*, November, 1892.

ordinates for the corresponding time points represent the utility and disutility resulting from the corresponding unit of labor, we get the familiar utility and disutility curves *UU′* and *DD′*, respectively, which intersect at the point *P*. It is evident that the value or marginal utility of the day's product—*ABPE*—is equal *in amount* to the marginal disutility of the day's product, both being equal to the number of units of labor time consumed when *BP* is taken as the unit of utility and disutility. Hence Professor Clark's conclusion that at any given time, and in respect to any isolated laborer the value of his product is most easily measured by the amount of labor time expended in producing it.




But value, Professor Clark holds, is a social phenomenon.[1] In modern life no one produces for his own consumption. We obtain the goods we wish to consume by producing others which we exchange for them. A produces $c_1$, B produces $c_2$, C produces $c_3$. $c_1$ is distributed among A, B, and C in return for their work, or the products of their labor. The value of $c_1$ is not measured by the time it takes A to produce it, but by the time it takes A, B, and C to produce the goods which are exchanged for it. The same argument applies to every product of the society; the value of each commodity is best measured by the social or aggregate labor time spent in acquiring it. "The standard for measuring it (value) is the sacrifice in final periods of labor, entailed on society in acquiring it." [2]

Professor Clark's standard is evidently the old labor standard made up of all kinds of labor in the relative proportions in which they are expended. The defects of this standard have already been considered. In the first place labor is like other commodities in respect to the fact that at different times the various kinds of labor are purchased in different amounts. In the second place, as is shown by the dotted lines in the last dia-

[1] See *New Englander*, Vol. IV (n. s.), p. 457.

[2] *Yale Review*, November, 1892, p. 272.

gram, the unit of disutility, or the length of the day's work, must, *ex hypothesei*, vary when the productivity of labor changes. Thus, as is shown in the diagram, if the utility curve changes from $UU'$ to $U''\,U'''$, we will have a new unit of disutility (and of marginal utility), $B'P'$. The relation of $BP$ to $B'P'$ is unknown —is, in fact, the ratio we seek. However, what is more likely to happen in such a case, is a change in the relation of utility and disutility. The standard of life will change, the laborer will demand more utility per unit of disutility; he will perhaps continue to work $AB$ hours, but will demand for the marginal unit of disutility a larger amount of utility, $BD''$, than before.

So obvious are these facts that I question whether Professor Clark proposes his standard as one suitable for the actual measurement of variations in value from time to time. From the present standpoint, however, he demonstrates nothing more than the proposition that at any given time a certain quantity of labor may be used as an ultimate standard of value—a proposition that has been repeated hundreds of times in the phrase "any commodity will serve as a measure of value at a given time and place."

There is, consequently, little assistance to be derived from the formulæ of Professor Clark and Dr. Lehr. The one is a weighted labor standard, the other a new variety of the compromise consumption standards noted on page 7 of the preceding article. Both have been shown to be theoretically defective, and both require statistical data which it is almost impossible to secure.

The utility standard proposed in this paper, however, requires only a few commodities, the *per capita* consumption of which is approximately the same in $T'$ and $T''$, and manifestly rests upon the proposition that the value or utility of a commodity is constant as the quantity consumed is constant. An apparent objection to this proposition is found in the fact that the utility of a commodity is a function, not only of its own consumption quantity, but in some cases is also a function of the quantities of other commodities. Thus, if an individual consume $X$ units of

tea and $Y$ units of coffee in $T'$, while in $T''$ he consumes $X$ units of tea and $NY$ units of coffee, it is possible that the utility of tea would not be exactly the same in $T'$ as in $T''$.

In the above illustration it is evident that the error caused by regarding tea as invariable in $T'$ and $T''$ would be insignificant. That part of the value of tea which is dependent upon the quantity of coffee consumed is, under ordinary circumstances, a quantity of the second order which may be safely neglected. But in certain other commodities this interdependence of values cannot be neglected. Some goods are practically interchangeable. "When herring are dear the people buy sprats." They may not like sprats so well, but both satisfy hunger.

In consequence, it is not wholly true that a quantitative change in the consumption of a commodity is a certain indication of a change in its utility. On the other hand, it is more nearly true that fixity in quantity indicates fixity of utility, because when the quantity of one "complementary" or "substitutionary" good changes, the quantity of the other also changes. In all such instances a theoretical remedy lies in the exclusion of both commodities from the standard, when the quantity of either changes. The practical remedy, however, lies in the fact that the utility of a great majority of commodities is so little dependent upon the quantity of other commodities that this factor is negligible.

In another and more vital sense the value of any commodity may be said to be a function of the quantity of other goods. Excluding substitutes, complementary and competing goods it is evident that the utility of any commodity depends upon the quantity of other commodities, because of the mobility of labor and capital. If, through invention or discovery, labor and capital become more productive in one industry, while the conditions of production in other industries experience no variations of their own, labor and capital will flow from the first to the other industries. The effect of the invention or discovery will thus be spread over the whole industrial mechanism, increasing the output in all lines and decreasing the utility of every commodity.

But changes in the conditions of production are not all of one kind. Opposed to the march of invention is the law of decreasing returns. Each progressive step in any industry operates to send some of its labor into other industries, thus increasing the supply of all commodities. Every retrogressive step has an opposite effect. Two general forces are thus continually being propagated. If these forces are equal as well as opposite, they will neutralize each other, leaving those industries which have experienced no proper variations unaffected. If one of these forces be greater, producing a resultant general force, we may expect to see this resultant neutralized in those industries which have experienced an equal and opposite variation of their own. Such industries will furnish the standard commodities desired. It is not necessary that the quantity of such goods should be the same throughout the whole interval $T' - T''$. It is only necessary that the quantities be the same in $T'$ as in $T''$. Neither is it essential that the standard should always consist of the same commodities. In comparing $T'$ and $T''$ we may use the commodities $c_1$, $c_3$, $c_5$; while in comparing $T''$ and $T'''$, the commodities $c_2$, $c_1$, $c_4$ may be used.

Looking now to the consumption of the individual, we find a similar interdependence of consumption quantities. When the price of the commodity $c_1$ rises, the quantity consumed immediately falls. But this will not, in most cases, be the only effect; the quantities of other commodities will also fall. If $c_1$ be some "indispensable" commodity—a medicinal article, for instance—the fall in the quantity of $c_1$ may be small, while the quantities of other goods—*e. g.*, certain luxuries—may decrease in a greater degree. $c_1$ having become more valuable, other goods must become more valuable. In accordance with a principle which may be called the *mobility of consumption*, the utility of a good is a function of the quantities of all other goods. As in the preceding case, variations are not all of one kind between $T'$ and $T''$. Positive price variations set in action general forces which are neutralized in certain lines of consumption by the general forces propagated by negative price variations.

In such lines of consumption we find the standard commodities desired. It is interesting to note that the mobility of consumption is practically instantaneous in its operation. The flow of labor and capital from one industry to another is obstructed in many ways. But no similar obstructions are found in expending money for consumption.

But production and consumption are, of course, interdependent. Whether or not a good is consumed in the same quantities in $T'$ and $T''$ depends upon the interaction of two forces: the change in the price of the good, and the change in the income of society. Prices may rise uniformly. In this case, if incomes rise in the same ratio, commodities may be consumed in the same quantities in $T'$ and $T''$. What actually happens, however, is that some prices rise and others fall. Whether the standard commodities will come from the group whose prices rise or the group whose prices fall depends upon the movement of income between $T'$ and $T''$. If incomes have risen in $T''$ the society will be enabled to consume the usual quantities of the goods whose prices have increased. If incomes have fallen, only those goods whose prices have fallen will continue to be consumed in the old quantities. The forces which influence the value of money are various, but their resultant is measured in the prices of those goods which are consumed in the same quantities.

The proposed standard, then, is, without aiming to be so, a logical compromise between the labor and consumption standards. Böhm-Bawerk has mentioned this attribute as a prime requisite of the ultimate standard of value, and its importance is manifest.[1] The consumption and labor standards, and their respective defendants, are not wholly wrong; the standards are based upon important half-truths, and the defendant of each feels in his cause a measure of justice. The trouble is that the defendant of the consumption standard never recognizes the semi-justice of his opponent's position, and *vice versa.* Men like David A. Wells assume the validity of the labor standard,

[1] *Annals of the American Academy* Vol. V., Pt. 1, p. 208.

construct strong and convincing arguments from this standpoint, and then wonder why they make no impression upon the vast number of people who have no idea of, or at least no sympathy with, any other than the consumption or tabular standard.[1] The reason lies in the simple fact that the value of money depends upon both the level of prices and the size of the income. Whether or not money was more valuable in 1896 than in 1873 depends upon whether the effect of the fall in prices was great enough to offset the effect of the increase in wages.

The standard of value proposed here rests upon the marginal utility theory of value, and, although described simply as a standard of value, is in its first interpretation a standard of utility. The nature of the unit or standard of utility has been the subject of considerable speculation.[2] Some writers seem to have assumed that it would necessarily be a unit of sensation. Others have raised unfortunate distinctions between standards of marginal and total or absolute utility. Almost all of them have assumed that the standard of utility was hopelessly impracticable. An examination of the more important of these objections will serve to show that the standard of utility is not only not impracticable, but that it really constitutes the only practicable standard by which changes in the value of money can be measured.

In the discussion of the proper standard of deferred payments, carried on several years ago between Professor Ross and Dr. Merriam,[3] the impression was created that, since value is measured by marginal utility the correct standard of value would be a standard of marginal utility as distinguished from a standard of total or absolute utility.

This distinction arises from a misconception of the nature of utility which is mischievous, not only because of its prevalence,

[1] See WELLS, *Recent Economic Changes*, chap. v.

[2] ROSS, *Annals of the American Academy*, November 1892, and November, 1893; MERRIAM, *ibid.*, January, 1893; FETTER, *ibid.*, May, 1895; MENGER, *Revue d'Economie Politique*, February, 1892.

[3] *Annals of the American Academy*, November, 189 ; January, 1893; November, 1893.

but because of the fact that a majority of persons, if called upon to choose between a standard of total utility and one of marginal utility, would without hesitation prefer the former. It is, therefore, important to show that the differentiation of marginal from total utility in the standard of value is impossible, and that, in consequence, the standard proposed here is not open to the objections formulated by Professor Ross and Dr. Fetter[1] against the theoretical standard defended by Mr. Merriam.

In order to gain any definite conception of a value constant in different epochs, it is necessary to make value equivalent to that property of goods which has been called "utility," "desiredness," "ophelimity," "real value," etc., and for the purpose of measuring variations in value it is necessary to postulate that the amounts of this property contained by various goods are proportional to their respective prices. This fact is most frequently expressed in the simple statement that at any given time marginal utilities are proportional to prices. It is usually deduced as follows: The last dollar spent for every commodity must yield the same amount of utility, or I would buy more of some commodities and less of others. Thus, if the last dollar's worth of bread yields me less utility that the last dollar's worth of meat, I will buy more meat and less bread until the equilibrium be restored. Hence, marginal utility of $\frac{1}{p_1}$ units of $c_1 =$ marginal utility of $\frac{1}{p_2}$ units of $c_2$. This expression is then expanded so as to read:

$$\frac{\text{mar. ut. of 1 unit of } c_1}{\text{mar. ut. of 1 unit of } c_2} = \frac{p_1}{p_2}\,.$$

But this expression is, by admission, only approximately true. By the law of variable utility the utility of one unit of $c_1$ is not $p_1$ times the utility of $\frac{1}{p_1}c_1$. Consequently, writers have been careful to say that this law holds only when $c_1$ and $c_2$ are very small, or, in other words, that the law is strictly true only for differential increments, $dc_1$ $dc_2$, etc.

[1] *Annals*, November, 1893, and May, 1895.

The law which states that marginal utilities are proportional to prices is, according to this analysis, evidently open to three criticisms which have been frequently urged against it. It is asserted: (1) that it postulates the infinite divisibility of commodities; (2) that it holds only for infinitesimal marginal increments; (3) that a fixed amount of marginal utility is an empty philosophical vagary, impractical and unsuited (even if practical) to act as a standard of value.

These criticisms have originated in a defective or, at least, an inadequate analysis of consumption. In order to test their validity it is desirable to examine more closely into the nature of the utility with which we are concerned.

In speaking of utility it is necessary to limit the connotation of the word by reference to a definite commodity, and most important of all, a definite time epoch. We must think of the utility of the commodity $c_1$ in the epoch $T$. The omission of the time element in discussions of utility is largely responsible for the objections noted.

In order to look more closely into the process of consumption, a simple illustration may be employed. Suppose an individual to heat his house with a ton of coals per month. The commodity unit here will be the ton of coals; the time unit the month; and the utility unit the total utility afforded by the ton of coals. We may further assume that the ton is divided into a number of uniform subdivisions, say bushels. The following inferences may be drawn:

1. That irrespective of the actual amounts of utility afforded by the several bushels the individual will regard the utility afforded by each bushel as the same. Either at the beginning or at the end of the month each bushel will appear equally important, though it may be noted that we are here wholly beyond the domain of exchange value.

2. That so far as there is any law discernible the actual process of consumption tends to follow the process of thought. Notwithstanding the gradual satiability of want, equal subdivisions of the unit of commodity tend to yield uniform amounts

of utility. *Cæteris paribus*, the individual will distribute the time and divide the commodity so that equal portions of the commodity will yield like amounts of utility, because in this way the maximum amount of satisfaction will be obtained. Shorten the time unit to the dinner hour, include only one class of commodities—food—think of utility as the mere power of satisfying hunger, and we obtain the concept of utility that has fathered the above criticisms and inveigled economics into the hopeless search for a unit of sensation. A definite quantity of a commodity designed to be consumed in a definite period of time will be consumed in a way such that equal subdivisions of the commodity will tend to yield uniform amounts of utility.

Utilities, then, are proportional to prices, and the relation holds good for total utility and actual commodities as well as for marginal utility and infinitesimal increments. The individual regulates his purchases with reference to a general level of possible utility, which is dependent upon the amount of his income. Knowing his total income he lays out fixed amounts for the different goods. He knows that he can purchase so much $c_1$ so much $c_2$, so much $c_3$. The more he buys of $c_1$, the less satisfaction per unit will be receive, and the same is true of $c_1$, $c_2$, $c_3$. But each subdivision of $c_1$ does not yield less satisfaction than every preceding subdivision. Each succeeding increment of $c_1$ may bring down the amount of utility which would be rendered by preceding subdivisions, if the last increment were not consumed. But the latter statement is by no means equivalent to the former statement. Taking the annual consumption of a commodity as the unit, it is clear that we cannot treat the first increment or subdivision consumed in January as more useful, more desirable, or more valuable than the last increment consumed in December.

It follows also that the differentiation of a standard of marginal utility from the standard of total utility is impossible. We imagine a case in which a man has only one loaf of bread a week. We deduce from this the correct conclusion that in this event the utility of the loaf would be practically infinite. We

then imagine the man to have ten loaves per week and from the second case deduce the incorrect result that the utility of the first loaf is still infinitely greater than the utility of the tenth loaf. Such an inference is entirely unwarranted. It assigns to the first loaf of ten the utility which it would possess if the individual had only one loaf. The result in our problem is to furnish us a standard of potential utility with which we have no concern. There is no difference between a standard of marginal and a standard of total utility. The distinction is between utility and potential utility.

As there is no useful distinction between marginal and total utility, there would seem to be no distinction between utility and value, unless the basis of differentiation be those moral or non-economic criteria according to which we adjudge that a quart of whisky possesses less utility than a loaf of bread, or a bushel of coals more utility than a diamond. With such distinctions the theory of value has no concern. Accordingly, I see no reason to employ any other term than the simple word value. I dispense with the word utility the more willingly because many prominent writers (with a verbal justification, I must admit) deny that two physically dissimilar goods can be said to possess like amounts of utility. The utility afforded by a dollar's worth of candy and that afforded by a dollar's worth of quinine, they say, are incommensurable. Terms are not material. If the utility afforded by the two goods is not the same, the economist has nothing to do with utility. His subject-matter is that property which two commodities may be affirmed to possess in like amounts when the individual desires neither to the exclusion of the other.[1] If one finds his pathway blocked by a post, it is useless to argue; the sensible thing is to avoid the post.

Before closing it is necessary to say a few words of the practical application and advantages of the proposed standard.

At the outset it must, of course, be admitted that the absolute requirements of this standard cannot be satisfied any more

[1] See FISHER, *Mathematical Investigations*, pp. 3–24; *cf.* SHERWOOD, "The Philosophical Basis of Economics," *Publications of the American Academy*, No. 209; and MARSHALL, *Fortnightly Review*, April, 1876, pp. 596, 597.

than the absolute requirements of any physical measurement can be satisfied. The reasons for this are obvious. While we can reasonably hope for the necessary statistics of consumption, as soon as the government becomes awake to the importance of such data, we do not at present possess them. On the other hand, if such statistics were available, they would seldom or never show a commodity whose per capita consumption was exactly the same in $T'$ and $T''$. We should be forced to include commodities whose quantities had varied a little, say 5 per cent., in the interval; and this would introduce an element of error which could only be removed by the use of a mean. At some point in the investigation we must "trust to luck," average the returns in such a way as to make the errors mutually destructive, and test the result by the criteria of the science of probability.

But at just what point we shall begin to trust to luck is a question of the greatest practical importance. If we attempt to ascertain the mean human stature of a nation, to borrow an illustration from Dr. Venn, it is very evident that we must not only measure an immense number of people,[1] but we must select these people judiciously. It will not do to take most of the measurements in the cities and a very few from the rural districts, or all from the East and none from the West, because this will influence the mean type which we are trying to elicit. Moreover, it is plain that this mean type is a very elusive affair. At bottom, it seems largely a creature of definition. We can modify it by confining our investigation to a limited area. It measures no known force or set of forces. It is useful chiefly for purposes of comparison; it is valuable, for instance, to know that on the average Englishmen are so many inches taller than Frenchmen.

On the other hand, to borrow another illustration from Dr. Venn,[2] almost verbatim, suppose that a man had been firing at a small mark on a wall, that the mark had subsequently been

[1] VENN, *Logic of Chance*, chap. 2: "It need hardly be insisted upon that the interest and significance of such investigations as these are almost entirely dependent upon the statistics being very extensive," p. 24.

[2] *Ibid.*, chap. 19, § 2.

removed, and we were asked to guess the position of the mark from the arrangement of the shot-marks. It is evident here that, unless some regular force, such as the wind, had exerted an unvarying influence upon the shots, we should be able to locate the mark with all necessary precision from a comparatively small number of shot-marks. The fact that we had been aiming at a fixed object would simplify the matter greatly. And until the position of the wafer had been estimated by the appropriate average, each shot would be an equally valid indication of the position desired.

The relation between the two problems just illustrated is very similar to the relation between the average of all price variations obtainable and the average of those commodities whose consumption quantities have not varied. The former requires an immense number of prices, which must be representative as well as independent, and when the variations of these prices have been averaged the meaning and importance of the result are uncertain. As in the first illustration, its chief utility is probably comparative; that is, it would possibly furnish a valid method of *comparing* the general movement of prices in two intervals of time, or in two countries for the same interval of time. This is a very different thing, however, from a *measurement* of change or difference in the value of money between two epochs or places.

The index number suggested here, however, partakes of the objectivity of a physical measurement, in which each of the returns is a slightly inaccurate measurement of the same thing. If the theory be correct upon which this index number is based, our result will be more exact the more exclusively we confine ourselves to those commodities which are consumed in exactly the same quantities in $T'$ and $T''$. With a standard approaching this condition within, say 5 per cent., as suggested, the margin of probable errors would be small, although it might happen that one or two of the variations diverged greatly from the rest, and for this as well as other important reasons it would be advisable to use the median. But, because of the "objectivity" of the measurement and the consequent restriction of the errors,

there would not be the same necessity for a large number of prices that there is in the ordinary index number, and the accuracy of the result would not be so largely dependent upon the selection of a representative list of independent price variations.

In the ordinary consumption index number, the importance of selecting a truly representative list of independent variations is as great as the task is difficult. In the actual computation of index numbers this difficulty expresses itself in the concrete question: Ought we to take a few very important commodities whose price variations are wholly distinct, or should we include as many prices as possible, placing more emphasis upon quantity than upon importance and independence?[1] "Common sense' furnishes no guide here, as it is extremely desirable both that the commodities should be important and the price variations numerous, and, moreover, "common sense" is a notoriously poor guide in delicate quantitative questions. The importance of the problem is shown by the fact that if we throw the index numbers of Falkner, Soetbeer, and Sauerbeck upon the same basis, the first (based upon 223 articles) will in practically every year be found higher than the second (114 articles), and the second higher than the third (45 articles). While the grave discrepancies between these index numbers is probably due in a large degree to actual differences in the variation of money in the United States, Germany, and England, they are probably due in a greater degree to the number and character of the commodities covered by the several measurements.[2]

In the index number proposed here, however, this question would not arise, as the importance of the commodity in consumption would have little or no weight. The change in the price of pepper is just as good an indication of the variation in the value of money as the change in the price of wheat. Neither would it be fatal to include a disproportionate number of prices from one general field of industry. If, for instance, five equally

[1] See the interesting debate upon this question between Mr. Sauerbeck and Professor Pierson, in the fifth volume of the *Economic Journal.*

[2] See ALDRICH, *Report upon Wholesale Prices, Wages, and Transportation*, Part I, p. 256 *et passim.*

expert surveyors measure the length of a river with the same instruments, and one of the surveyors make twice as many measurements as any other, no grave error could be introduced by taking an average of the returns without reference to the particular individual who made them. Moreover, no weights would be required and our result would have a definite meaning in itself; it would be the best measurement we could make of the change in the value of money, not an ambiguous "average variation of prices."

In conclusion it may be added that the theory upon which this standard is based furnishes a test of the direction of the variation in the value of money, which may prove not without interest to those impartial investigators who have hesitated between the claims of the labor and consumption standards and who, in consequence, have been uncertain whether the value of money rose or fell between 1860 and 1891, to take the standard and final years of the Aldrich investigation, for example. This test is found in the consumption of those commodities whose prices were the same in 1860 and 1891. If a dollar purchased $c_1$, $c_2$, $c_n$, etc., in both 1860 and 1891, then the value of each of these commodities must have varied exactly as the value of money varied between 1860 and 1891, and the direction of this variation will be shown by the change in the consumption of these articles. If the consumption of these articles increased, the value of money fell in the interval; if the consumption decreased, the value of money rose. *A fortiori*, if the consumption of those articles whose prices have risen increased, the value of money must have fallen.

Table II, Part I, of the Aldrich report on *Wholesale Prices, Wages, and Transportation* shows that the prices of the following articles were the same in 1860 and 1891: potatoes, milk, soda crackers, broadcloths, coal (anthracite, pea), chestnut lumber in the log, wooden tubs, Ontario starch. The following articles were from 2 to 10 per cent. higher in 1891 than in 1860: lamb, beef (loins), beef (ribs), rye flour, Sumatra pepper (whole), eggs, anthracite coal (egg and stove), sugar of lead (white),

linseed oil. To base a verdict concerning the change in the value of money upon these data, without accurate statistics of consumption, is of course out of the question, but in connection with the more extensive price lists from which they are taken and the general belief that the consumption of meat and coal, for instance, is increasing, they furnish some reason for the belief that in the last half century, to use round figures, the value of money has fallen rather than increased, or, in other words, that the increase in wages has more than offset the fall in prices.

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